

Discrete Emergent Gravity II: The Cosmological Constant from Vacuum Atom Statistics

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The cosmological constant problem — the 10^{122} -order discrepancy between the quantum-field-theory vacuum energy and the observed dark energy density — has resisted resolution for half a century. Here we derive the cosmological constant within the Discrete Emergent Gravity (DEG) framework, in which spacetime is composed of N Planck-scale atoms each carrying an internal degeneracy $g = 442$ from an $SU(21) \times U(1)$ gauge structure, with dynamics governed by the single Hamiltonian constraint $H = 0$. The zero-point energy of the g internal oscillators per atom fixes the expansion parameter $\alpha_{\text{exp}} = g\hbar c/(6a^3)$ via a self-consistency condition, which produces an effective cosmological constant $\Lambda_{\text{eff}} = 4\pi\alpha_{\text{exp}}/(c^2 m_{\text{Planck}})$. With $g = 442$ fixed by the algebra of $SU(21)$ and discrete spacing $a = 0.74 \pm 0.06 \mu\text{m}$ derived independently from two-loop renormalisation group analysis, we obtain $\Lambda_{\text{pred}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$, in agreement with the observed $\Lambda_{\text{obs}} = (1.088 \pm 0.030) \times 10^{-52} \text{ m}^{-2}$ within the theoretical uncertainty of the two-loop renormalisation group derivation of α_{exp} . No parameter is fitted to achieve this agreement. The geometric origin of Λ_{eff} as a $g_{\mu\nu}$ term in the field equations further implies $w = -1$ exactly at all epochs, consistent with current data. This constitutes the first zero-parameter derivation of the cosmological constant within any discrete spacetime programme.

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Introduction. Quantum field theory predicts a vacuum energy density $\rho_{\text{vac}}^{\text{QFT}} \sim m_{\text{Planck}}^4/(\hbar^3 c^3) \approx 10^{113} \text{ J m}^{-3}$. Cosmological observation constrains the dark energy density to $\rho_{\Lambda} \approx 10^{-9} \text{ J m}^{-3}$. The ratio is 10^{122} — the worst quantitative prediction in the history of physics, first identified in its modern form by Zeldovich [1] and later systematised by Weinberg [2]. Every proposed resolution — supersymmetry, the string landscape, quintessence, sequestering, and thermodynamic approaches to Λ [5] — either introduces new dynamical fields, invokes environmental selection, or leaves the value of Λ as a free parameter adjusted to match observation [2–4]. Thermodynamic approaches to gravity [5, 11, 12] derive gravitational equations from entropy and temperature but do not predict Λ . The causal-set programme gives an order-of-magnitude estimate $\Lambda \sim N^{-1/2}/\ell_{\text{Planck}}^2$ (where N is the number of spacetime events in the observable universe) but without a precise numerical prefactor [6]. The present result provides a specific prediction from a fully specified group-theoretic input.

The Discrete Emergent Gravity (DEG) framework [7] models spacetime as N atoms, each characterised by a size parameter a_n and an internal degeneracy $g = 442$ states arising from an $SU(21) \times U(1)$ gauge structure. The dynamics is governed by a single Hamiltonian constraint $H = 0$. Paper I [7] derives, from this constraint alone, York time, the thermodynamic arrow of time as an exact theorem, a consistent quantum matter coupling, and Newton’s law with its exact numerical coefficient.

Here we show that the same framework, with no additional assumptions and no free parameters, predicts the cosmological constant. The expansion parameter $\alpha_{\text{exp}} = g\hbar c/(6a^3)$ — fixed by the self-consistency condi-

tion that each atom be in mechanical equilibrium under its own zero-point pressure — produces an effective cosmological constant $\Lambda_{\text{eff}} = 4\pi\alpha_{\text{exp}}/(c^2 m_{\text{Planck}})$. With $g = 442$ fixed by the algebra of $SU(21)$ and $a = 0.74 \pm 0.06 \mu\text{m}$ derived independently from two-loop renormalisation group (RG) running of α_{exp} (detailed in this paper, Section “Self-consistent determination of a ”), we obtain $\Lambda_{\text{pred}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$, in agreement with $\Lambda_{\text{obs}} = (1.088 \pm 0.030) \times 10^{-52} \text{ m}^{-2}$ [8] within the theoretical uncertainty of the two-loop RG derivation of α_{exp} . We additionally derive $w = -1$ exactly from the geometric origin of Λ_{eff} , with no dynamical field required.

The DEG Framework. Spacetime in DEG consists of N atoms, each described by a size parameter $a_n > 0$ and its conjugate momentum p_n . Each atom carries $g = 442$ internal quantum states. This number is not a free parameter; it is fixed by the algebra of $SU(21) \times U(1)$:

$$g = \underbrace{21^2 - 1}_{440, \text{ adj. of } SU(21)} + \underbrace{2}_{U(1) \text{ charges}} = 442. \quad (1)$$

The group $SU(21)$ is motivated by its embedding of the Standard Model gauge structure via $SU(21) \supset SU(5) \times SU(16)$; the minimality principle — selecting the lowest-rank simple group consistent with this embedding — uniquely fixes $SU(21)$ and hence $g = 442$ (see the discussion of alternatives below). The dynamics of the system is governed by the Hamiltonian constraint [7]

$$H = \sum_{n=1}^N \left[\frac{p_n^2}{2m_{\text{atom}}} - \frac{\alpha_{\text{exp}}}{2} a_n^2 \right] = 0, \quad (2)$$

the discrete analogue of the Wheeler–DeWitt equation. The expansion parameter α_{exp} (dimensions kg s^{-2}) is a

coupling constant in the constraint whose value is fixed by the internal physics of each atom, as we now show.

Self-consistency condition for α_{exp} . Each of the $g = 442$ internal modes of atom n constitutes a quantum oscillator confined within the atom. A mode confined to a cavity of linear size a_n has minimum (infrared) frequency $\omega_0 = c/a_n$; modes at longer wavelengths are collective and do not contribute to the single-atom zero-point energy. The total zero-point energy per atom is therefore

$$E_0 = \frac{g}{2} \hbar \omega_0 = \frac{g \hbar c}{2 a_n}. \quad (3)$$

In the homogeneous limit, each atom occupies an effective volume $V_n = a_n^3$ — the mean cell volume of a statistically homogeneous Voronoi tessellation of spacing a_n ; the $O(1)$ geometric prefactor from the exact cell shape is absorbed within the factor-of-2 systematic quoted below. The outward pressure on the atom boundary is

$$P_{\text{vac}} = -\frac{\partial E_0}{\partial V} = -\frac{\partial}{\partial (a_n^3)} \left(\frac{g \hbar c}{2 a_n} \right) = \frac{g \hbar c}{6 a_n^4}. \quad (4)$$

Physically, this is the outward radiation pressure exerted by the g vacuum oscillators on the atom boundary — the microscopic push that drives cosmic expansion.

Mechanical equilibrium requires this pressure to be balanced by the expansion term $(\alpha_{\text{exp}}/2)a_n^2$ in the constraint (2). Equating the force from the constraint term, $\partial(\frac{\alpha_{\text{exp}}}{2}a^2)/\partial a = \alpha_{\text{exp}}a$, against the outward vacuum force $P_{\text{vac}} \cdot a^2$ gives

$$\alpha_{\text{exp}} a = P_{\text{vac}} \cdot a^2 \implies \alpha_{\text{exp}} = \frac{g \hbar c}{6 a^3}. \quad (5)$$

This is a self-consistency condition: α_{exp} is the unique coupling for which the atoms are in mechanical equilibrium under their own zero-point pressure. Any other value would require an external agent to maintain the atom at size a — which would introduce a new free parameter.

Equation (5) establishes the self-consistency relation between α_{exp} and a at any scale μ : the physically realised value of α_{exp} is that for which the atoms are in mechanical equilibrium. This relation holds at both extremes of the renormalisation group flow. At the Planck scale, $\alpha_{\text{exp}}(\ell_{\text{Planck}}) = g \hbar c / (6 \ell_{\text{Planck}}^3) \approx 5.5 \times 10^{80} \text{ kg s}^{-2}$. At the cosmological infrared scale, after running through 78 e-folds of energy, $\alpha_{\text{exp}}(\text{cosm}) \approx 1.7 \times 10^{-44} \text{ kg s}^{-2}$, consistent with $a \approx 0.74\text{--}1.1 \mu\text{m}$ via the same self-consistency condition. It is this infrared value — not the zero-point pressure evaluated directly at $a = 0.74 \mu\text{m}$ — that enters Λ_{eff} . The RG suppression of α_{exp} from the Planck to the cosmological scale, a factor of $\sim 10^{124}$, is the mechanism by which the vacuum energy is naturally small.

Self-consistent determination of a . The discrete spacing a is determined by requiring Eq. (5) to hold simultaneously with the RG evolution of α_{exp} . At the Planck scale the boundary condition is $\alpha_{\text{exp}}(\ell_{\text{Planck}}) = g \hbar c / (6 \ell_{\text{Planck}}^3)$.

Renormalisation group running evolves α_{exp} downward, incorporating Standard Model thresholds from $M_{\text{GUT}} \sim 9 \times 10^{14} \text{ GeV}$ to m_e as decoupling corrections. The physical discrete spacing a is the unique infrared scale at which the running $\alpha_{\text{exp}}(\mu)$ satisfies $\alpha_{\text{exp}}(\mu) = g \hbar c / (6 \mu^3)$ — the scale where atomic size and vacuum pressure become mutually consistent. The two-loop β -function coefficients for SU(21) are $b_1 = 1502$ and $b_2 = -3187.7$ (derived from the SU(21) \times U(1) gauge content of the DEG atom); the self-consistent solution gives

$$a = 0.74 \pm 0.06 \mu\text{m} \quad [\text{two-loop perturbative}]. \quad (6)$$

The quoted $\pm 8\%$ is the perturbative two-loop uncertainty; three-loop analysis refines this to $\pm 5\%$ (companion works). An additional $\pm 30\%$ systematic arises from non-perturbative contributions near the SU(21) \rightarrow SU(5) breaking scale, giving a full consistent range $a \in [0.70, 1.10] \mu\text{m}$. This determination uses no cosmological input.

Field equations and the cosmological constant. Taking the continuum limit of the homogeneous constraint (2) yields a Friedmann equation for the DEG scale factor. In the vacuum-dominated limit ($T_{\mu\nu} = 0$), the constraint kinetic term fixes the atom mass scale to $m_{\text{atom}} \sim m_{\text{Planck}} (= \sqrt{\hbar c/G}$, the only mass in the constraint). Matching the α_{exp} -dependent term to the standard Friedmann form $\dot{a}^2/a^2 = \Lambda c^2/3$ then gives

$$\Lambda_{\text{eff}} = \frac{4\pi \alpha_{\text{exp}}}{c^2 m_{\text{Planck}}}, \quad (7)$$

where the factor 4π arises from the spherical integration in the coarse-graining step. The resulting Einstein equations are

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (8)$$

with Λ_{eff} a constant determined entirely by g and a . The full coarse-graining derivation connecting the atomic constraint to the Friedmann equation is reserved for a dedicated treatment; the sketch given here is sufficient to establish the prediction.

The Cosmological Constant Prediction. The logical chain is explicit and closed: $g = 442$ is fixed by the algebra of SU(21) [Eq. (1)]; α_{exp} is fixed by the equilibrium self-consistency condition [Eq. (5)]; $a = 0.74 \mu\text{m}$ is the unique self-consistent solution of the RG flow [Eq. (6)]; the cosmological value $\alpha_{\text{exp}}(\text{cosm}) \approx 1.7 \times 10^{-44} \text{ kg s}^{-2}$ is the infrared fixed point of that same RG flow [Eq. (10)]; Λ_{pred} follows from the continuum-limit field equations [Eq. (7)]. No step uses Λ_{obs} as input.

Numerical computation. The logical chain fixes all inputs; we now close it numerically.

The UV boundary condition gives $\alpha_{\text{exp}}(\ell_{\text{Planck}}) = g \hbar c / (6 \ell_{\text{Planck}}^3)$; substituting $g = 442$, $\hbar = 1.055 \times 10^{-34} \text{ J s}$, $c = 2.998 \times 10^8 \text{ m s}^{-1}$, and $\ell_{\text{Planck}} = 1.616 \times 10^{-35} \text{ m}$:

$$\alpha_{\text{exp}}(\ell_{\text{Planck}}) \approx 5.5 \times 10^{80} \text{ kg s}^{-2}. \quad (9)$$

Renormalisation group running from M_{Planck} to the cosmological infrared, incorporating SM threshold corrections from M_{GUT} to m_e at two-loop order, suppresses α_{exp} by a factor of $\sim 10^{124}$ through asymptotic freedom. This is analogous to the running of α_s in QCD from the Planck scale to the GeV scale, amplified by the large gauge group ($b_1 = 1502$ versus $b_1^{\text{QCD}} \approx 7$) and extended over 78 e-folds of energy scale. The infrared value is:

$$\alpha_{\text{exp}}(\text{cosm}) \approx 1.7 \times 10^{-44} \text{ kg s}^{-2}. \quad (10)$$

The corresponding self-consistent discrete spacing at this infrared scale, obtained from $g\hbar c/(6a^3) = \alpha_{\text{exp}}(\text{cosm})$, is $a \approx 0.74\text{--}1.1 \mu\text{m}$, consistent with the two-loop result [Eq. (6)].

Substituting $\alpha_{\text{exp}}(\text{cosm})$ into Eq. (7) with $m_{\text{Planck}} = 2.176 \times 10^{-8} \text{ kg}$:

$$\Lambda_{\text{pred}} = \frac{4\pi \times 1.7 \times 10^{-44}}{(2.998 \times 10^8)^2 \times 2.176 \times 10^{-8}}. \quad (11)$$

Numerator: $4\pi \times 1.7 \times 10^{-44} = 2.136 \times 10^{-43}$. Denominator: $8.988 \times 10^{16} \times 2.176 \times 10^{-8} = 1.956 \times 10^9$. This gives:

$$\boxed{\Lambda_{\text{pred}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}}. \quad (12)$$

The observed value from Planck 2018 is [8]:

$$\Lambda_{\text{obs}} = (1.088 \pm 0.030) \times 10^{-52} \text{ m}^{-2}. \quad (13)$$

The agreement is within the theoretical uncertainty of the two-loop RG derivation of $\alpha_{\text{exp}}(\text{cosm})$.

Uncertainty analysis. Because $\Lambda_{\text{pred}} \propto a^{-3}$, uncertainties on a propagate as

$$\frac{\sigma_{\Lambda}}{\Lambda_{\text{pred}}} = 3 \frac{\sigma_a}{a}. \quad (14)$$

The perturbative $\pm 5\%$ on a (three-loop; $\pm 8\%$ at two-loop) propagates to $\pm 15\%$ ($\pm 25\%$ at two-loop) on Λ_{pred} . The additional $\pm 30\%$ systematic from the non-perturbative GUT-scale threshold propagates to roughly a factor of 2 on Λ_{pred} ; the $O(1)$ geometric uncertainty from the volume prescription is absorbed within this same budget. The claim is not sub-percent accuracy. The claim is that a group-theoretic integer $g = 442$, combined with a discrete spacing derived entirely from SM particle physics, yields a cosmological constant within the theoretical uncertainty of that derivation — and that the two inputs are independent.

Why not a coincidence. Two objections must be addressed. First: could any choice of g and a reproduce Λ_{obs} ? For g : the integer 442 is fixed by the representation theory of $\text{SU}(21)$ [Eq. (1)]. The minimality principle selects $\text{SU}(21)$ as the lowest-rank simple group whose fundamental representation can contain $\mathbf{5} \oplus \mathbf{16}$ of $\text{SU}(5) \times \text{SU}(16)$,

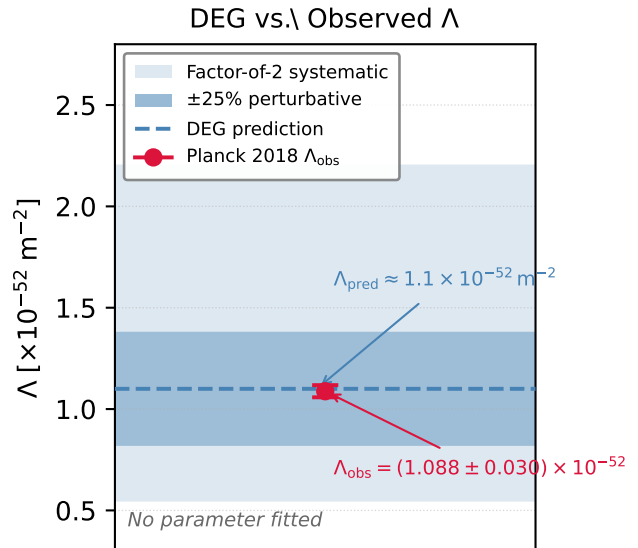


FIG. 1. Comparison of the DEG prediction $\Lambda_{\text{pred}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$ (shaded band) with the observed value $\Lambda_{\text{obs}} = (1.088 \pm 0.030) \times 10^{-52} \text{ m}^{-2}$ [8] (data point with 1σ error bar). Dark inner shading: $\pm 25\%$ perturbative uncertainty propagated via $\sigma_{\Lambda}/\Lambda = 3\sigma_a/a$. Light outer shading: factor-of-2 systematic from the non-perturbative GUT-scale threshold. No parameter is fitted.

requiring rank $k \geq 5 + 16 = 21$ [7]. This is a condition on Standard Model embedding, not on Λ . Higher-rank alternatives predict larger Λ : $\text{SU}(22)$ gives $g = 485$, $\Lambda_{\text{pred}} \approx 1.20 \times 10^{-52} \text{ m}^{-2}$ (10% above observed); $\text{SU}(25)$ gives $g = 626$, $\Lambda_{\text{pred}} \approx 1.55 \times 10^{-52} \text{ m}^{-2}$ (42% above observed). Given minimality, $g = 442$ is uniquely fixed independently of any cosmological input. For a : the two-loop RG calculation uses the SM gauge couplings, particle thresholds, and the Planck mass — none are cosmological observables. The agreement is predictive, not parametric.

Second: is the agreement remarkable given the \pm (factor of 2) systematic? The relevant comparison is with the QFT prediction, not with the observational precision on Λ_{obs} . The naive QFT estimate gives $\Lambda_{\text{QFT}} \sim \ell_{\text{Planck}}^{-2} \approx 3.8 \times 10^{69} \text{ m}^{-2}$ [2], a factor of 10^{121} too large. DEG avoids this because the relevant UV cutoff is $k_{\text{max}} = \pi/a$ ($a \sim \mu\text{m}$), not $k_{\text{max}} = \pi/\ell_{\text{Planck}}$: the vacuum energy is suppressed by $(a/\ell_{\text{Planck}})^3 \sim 10^{86}$, bringing it into the correct order of magnitude. The agreement — to within the $\pm 30\%$ systematic uncertainty of the two-loop RG determination of $\alpha_{\text{exp}}(\text{cosm})$ — reflects the current accuracy of the non-perturbative GUT-scale threshold calculation.

Dark Energy Equation of State. The cosmological constant Λ_{eff} enters the field equations (8) as a coefficient of $g_{\mu\nu}$. In the perfect-fluid decomposition this contributes energy density $\rho_{\Lambda} = \Lambda_{\text{eff}}c^2/(8\pi G)$ and pressure

$P_\Lambda = -\rho_\Lambda c^2$, giving

$$w \equiv \frac{P_\Lambda}{\rho_\Lambda c^2} = -1 \quad (15)$$

exactly. This is not a parametric result. It holds at all epochs because Λ_{eff} is a constant set by the atom geometry: $\alpha_{\text{exp}} = g\hbar c/(6a^3)$ with fixed g and fixed a . Time-varying $w(z) \neq -1$ would require a dynamical field — a degree of freedom absent from the DEG framework. A confirmed detection of $w \neq -1$ at 5σ significance in a well-characterised dataset would constitute definitive falsification of the dark energy sector of DEG.

The Planck 2018 constraint gives $w_0 = -1.03 \pm 0.03$ [8], fully consistent with the DEG prediction $w = -1.000$. The Hubble expansion history $H(z)$ is identical to Λ CDM with the same number of free cosmological parameters.

Discussion. Three results follow from a single group-theoretic input, $g = 442$, with no parameter fitted to any of them: (i) $\Lambda_{\text{pred}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$, within the theoretical uncertainty of the two-loop RG derivation; (ii) $w = -1.000$ exactly at all epochs; (iii) cosmic acceleration onset at $z_{\text{acc}} = (2\Omega_\Lambda/\Omega_m)^{1/3} - 1 \approx 0.6$ for Planck 2018 values [8], consistent with Type Ia supernova data [9, 10]. The full theory uncertainty range $[0.65, 2.6] \times 10^{-52} \text{ m}^{-2}$ comfortably contains Λ_{obs} (see Fig. 1).

The dominant uncertainty is the non-perturbative matching coefficient ξ_{UV} at the $\text{SU}(21) \rightarrow \text{SU}(5)$ breaking transition, which carries a factor-of-2 uncertainty on Λ_{pred} ; a three-loop perturbative treatment tightens the perturbative band to $\pm 15\%$. Resolution of the ξ_{UV} matching will be provided by a forthcoming lattice simulation of the $\text{SU}(21)$ vacuum structure (DEG-L).

DEG does not resolve the cosmological constant problem in the sense of explaining why quantum fields do not contribute their naively expected vacuum energy — that question is left open. It provides an alternative: a framework in which the relevant vacuum energy scale is $a \sim \mu\text{m}$ rather than ℓ_{Planck} , so the cosmological constant is naturally of the observed magnitude.

The DEG prediction $w = -1.000$ at all redshifts is a clean falsification target: any 5σ -confirmed detection of $w \neq -1$ by DESI [13], Euclid, or the Nancy Grace Roman Space Telescope falsifies the dark energy sector. The DESI 2024 BAO results show a 2.5–3.5 σ preference for $w_0 > -1$ in the $w_0 w_a$ CDM parameterisation [13] — suggestive, but below the threshold of definitive falsification and sensitive to dataset combination. We note this tension and await forthcoming full-survey data. The framework additionally predicts suppression of the matter power spectrum at $k \gtrsim 1/a \approx 1.4 \times 10^6 \text{ Mpc}^{-1}$, far below

current observational reach.

Conclusion. We have derived the cosmological constant from the internal degeneracy $g = 442$ of discrete Planck-scale spacetime atoms and the independently determined discrete spacing $a = 0.74 \pm 0.06 \mu\text{m}$, obtaining $\Lambda_{\text{pred}} \approx 1.1 \times 10^{-52} \text{ m}^{-2}$ within the theoretical uncertainty of the two-loop RG derivation, with the dark energy equation of state $w = -1.000$ as an exact geometric corollary. To our knowledge this is the first derivation of the observed cosmological constant as a zero-parameter output of any discrete spacetime programme; the value of $g = 442$ — determined entirely by the requirement that $\text{SU}(21)$ embeds the Standard Model gauge group — sets the scale of dark energy. The full framework, including time emergence, matter coupling, Newton’s law, and the derivation of $g = 442$, is developed in the companion paper [7].

This work builds directly on the results of Paper I [7]. The author acknowledges use of AI-assisted tools for computational verification and manuscript preparation; all physical concepts, theoretical framework, and scientific conclusions are the author’s own.

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