

Discrete Emergent Gravity I: Time, Matter Coupling, and the Derivation of Newton's Law

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Abstract

Two foundational problems in quantum gravity have resisted joint resolution in any discrete spacetime framework: the problem of time — how a meaningful time variable emerges from a timeless Hamiltonian constraint — and the matter coupling problem — how a quantum particle with a spread wavefunction couples consistently to a geometry with no continuous spatial labels. We address both within a single framework, the Discrete Emergent Gravity (DEG) programme, in which spacetime is composed of N Planck-scale atoms, each with size parameter a_n and an internal degeneracy $g = 442$ arising from an $SU(21) \times U(1)$ gauge structure, governed by the single Hamiltonian constraint $H = 0$.

From this constraint alone, we define York time as

$$\tau = \frac{1}{3N} \sum_{n=1}^N \ln \left(\frac{a_n}{\ell_{\text{Planck}}} \right),$$

and derive entropy by microstate counting: $S = k_B \sum_n [\ln g + 3 \ln(a_n/\ell_{\text{Planck}})]$. We prove, without any equilibrium assumption, that $dS/d\tau = 9k_B N > 0$, establishing the thermodynamic arrow of time as a theorem rather than an assumption. To couple quantum matter to discrete spacetime we introduce the quantum overlap $O_{mn} = \int_{\Omega_n} d^3r |\Psi_m(\mathbf{r})|^2$, giving a coupling Hamiltonian that is continuous, normalisation-preserving, and reduces exactly to the classical point-particle result for localised wavefunctions. Newton's law $F = GMm/r^2$ is then confirmed with the exact numerical coefficient via three mutually consistent verification methods, all grounded in a holographic screen entropy derived from DEG microstate counting; their agreement is a self-consistency check on that derivation.

In the companion paper (Paper II [45]), the same framework — with no additional assumptions and no fitted parameters — predicts the observed cosmological constant $\Lambda_{\text{obs}} = 1.1 \times 10^{-52} \text{ m}^{-2}$ within the theoretical uncertainty of the two-loop renormalisation group derivation. The cosmological constant problem has resisted resolution for decades; a zero-parameter derivation at this level of accuracy represents a qualitatively different kind of result, and Paper II develops it in full.

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I. INTRODUCTION

A. The problem of time in quantum gravity

At the quantum level, the universe does not evolve. The Wheeler–DeWitt equation [1, 2], obtained by applying the Dirac quantisation procedure to the gravitational Hamiltonian constraint, contains no time derivative: the wavefunction of the universe satisfies $\hat{H}\Psi = 0$, and the question of how — or whether — the universe changes becomes the *problem of time* [3, 4]. This is not a technical shortcoming of a particular quantisation scheme. It reflects the fundamental tension between quantum mechanics, in which time is an external parameter governing state evolution, and general relativity, in which time is part of the dynamical geometry subject to diffeomorphism invariance and carrying no preferred foliation.

Proposed resolutions fall into several families [5]. Barbour and collaborators [6–8] argue that time is fundamentally absent and only apparent, encoded relationally in the configuration of matter. Rovelli [9] proposes that time emerges from correlations between physical observables. Page and Wootters [12] construct an operational framework in which the clock reading of one subsystem serves as the time variable for another. Brown and Kuchař [15] identify matter degrees of freedom as natural clocks through a dust-field construction.

York [13, 14] introduced an alternative: the mean extrinsic curvature of a spacelike hypersurface, which he showed to be a globally well-defined, monotonically increasing parameter for expanding universes. This quantity, now called York time, has been studied in the quantum gravitational context by Gryb and Thébault [16] and Mercati [17].

The present work adopts York time as the emergent time variable and derives its key properties — gauge invariance, monotonicity, and connection to thermodynamics — from a fully discrete, Planck-scale description of spacetime. The derivation avoids the circularity identified by Kuchař [3]: York time is not presupposed as the evolution parameter but is computed as an output from a primary evolution parameter λ that appears in the Hamiltonian constraint. The distinction between λ and τ is central to the argument.

B. The matter coupling problem

Discrete approaches to quantum gravity — loop quantum gravity (LQG) [21, 22], causal dynamical triangulations (CDT) [23, 24], and causal sets [25, 26] — each construct a quantum

description of spacetime geometry from combinatorial or simplicial building blocks. They have achieved substantial results: area and volume quantisation in LQG, emergent de Sitter phases in CDT, and entropy-area relations in causal sets. What remains largely undeveloped in all three approaches is a systematic treatment of matter coupling at the level of individual discrete constituents: how does a quantum particle whose wavefunction is spread across many discrete cells couple to the geometry of those cells?

The difficulty is conceptual as well as technical. In a continuous spacetime, matter at position \mathbf{r} curves the geometry at \mathbf{r} . In a discrete spacetime, a quantum particle does not have a position — it has a wavefunction. Declaring that the particle “is in” the cell closest to its expectation value introduces discontinuities whenever the expectation value crosses a cell boundary, and fails entirely when the wavefunction is a superposition spread over many cells.

Section IV presents a resolution based on the quantum overlap O_{mn} , which distributes the matter’s influence across all cells in proportion to the probability of finding the particle in each cell. This is a direct application of the Born rule and carries no additional assumptions.

C. Our approach: discrete spacetime atoms

The DEG framework postulates that spacetime consists of N discrete atoms. Each atom n is characterised by a size parameter a_n (units of length), a conjugate momentum p_n , and $g = 442$ internal quantum states arising from an $SU(21) \times U(1)$ gauge structure. The atoms collectively tile three-dimensional space via a Voronoi tessellation.

The dynamics of the system is governed by a single Hamiltonian constraint $H = 0$, reflecting the absence of an external time in a closed gravitational system. This is the direct discrete analogue of the Wheeler–DeWitt equation. All subsequent results — time, entropy, matter coupling, and Newton’s law — are derived consequences of this single input.

The framework is not a discretisation of general relativity in the sense of putting a known continuum theory on a lattice. It is a *microscopic hypothesis*: if spacetime is built from atoms with the properties described above, these macroscopic structures emerge. The justification is empirical, not axiomatic. Newton’s law with the exact coefficient is one confirmation. The cosmological constant prediction (Paper II) is a second.

The general case of the DEG framework allows fully inhomogeneous distributions of atom sizes a_n . Homogeneity ($a_n = a$ for all n) is a special case that simplifies many formulae

and recovers the standard FLRW cosmology; we identify it as a special case rather than an assumption throughout.

D. Results and structure of this paper

This paper establishes three results.

Result 1 (time). From the Hamiltonian constraint alone, York time $\tau = (1/3N) \sum_n \ln(a_n/\ell_{\text{Planck}})$ emerges as a well-defined, gauge-invariant, monotonically increasing function of the atomic configuration. The entropy $S = k_B \sum_n [\ln g + 3 \ln(a_n/\ell_{\text{Planck}})]$, derived from microstate counting, satisfies $dS/d\tau = 9k_B N > 0$ exactly, with no equilibrium assumption.

Result 2 (matter coupling). The quantum overlap $O_{mn} = \int_{\Omega_n} d^3r |\Psi_m(\mathbf{r})|^2$, combined with the coupling Hamiltonian $H_{\text{coupling}} = -\sum_{mn} GM_m \rho_{\text{Planck}} a_n^2 O_{mn}$, provides a consistent, normalisation-preserving description of quantum matter in discrete spacetime. The classical limit is derived, not assumed.

Result 3 (Newton’s law). Newton’s law $F = GMm/r^2$ is confirmed with the exact numerical coefficient from three mutually consistent verification methods, all grounded in the holographic screen entropy derived from DEG microstate counting in Section V B. The agreement of the three methods is a self-consistency check on that derivation, not three independent confirmations of Newton’s law from independent inputs.

The paper is organised as follows. Section II introduces the DEG framework and its relation to prior work. Section III derives the time emergence and entropy results. Section IV develops matter-spacetime coupling. Section V verifies Newton’s law and carefully characterises the logical status of each step. Section VI discusses the results, limitations, and the load-bearing role of $g = 442$. Section VII concludes.

The three results presented here are not ends in themselves. They establish the DEG framework on rigorous footing. In the companion paper [45], we show that the same framework, with no additional assumptions and no fitted parameters, predicts the observed cosmological constant $\Lambda_{\text{obs}} = 1.1 \times 10^{-52} \text{ m}^{-2}$ within the theoretical uncertainty of the two-loop renormalisation group derivation. That prediction, and its comparison with observation, constitutes the primary empirical test of the DEG framework at the current stage of development.

II. THE DEG FRAMEWORK

A. Discrete spacetime atoms

We model spacetime as a collection of N elementary constituents, which we call *spacetime atoms*. Atom n is characterised by:

1. A size parameter $a_n > 0$ with dimensions of length, representing the characteristic linear extent of the region of space occupied by the atom. In the semiclassical regime, $a_n \gg \ell_{\text{Planck}}$.
2. A conjugate momentum p_n , with canonical Poisson bracket $\{a_n, p_m\} = \delta_{nm}$.
3. An internal quantum number $s_n \in \{1, \dots, g\}$, where $g = 442$. The internal states are degenerate in energy.

The evolution of the system is described by a *primary evolution parameter* λ , which is an arbitrary monotonic label — not physical time. The lapse function $N(\lambda)$ encodes the rate at which λ advances relative to proper time. Reparametrisation invariance requires that physical results be independent of the choice of $N(\lambda)$.

The dynamics is governed by the Hamiltonian constraint [33]

$$H = \sum_{n=1}^N \left[\frac{p_n^2}{2m_{\text{atom}}} - \frac{\alpha_{\text{exp}}}{2} a_n^2 \right] = 0, \quad (1)$$

where m_{atom} is an effective atom mass scale and α_{exp} is the expansion coupling constant with dimensions $[\alpha_{\text{exp}}] = \text{kg s}^{-2}$. Throughout this paper, $\ell_{\text{Planck}} = \sqrt{\hbar G/c^3} \approx 1.62 \times 10^{-35}$ m denotes the Planck length and $m_{\text{Planck}} = \sqrt{\hbar c/G} \approx 2.18 \times 10^{-8}$ kg the Planck mass, so that the Planck density is $\rho_{\text{Planck}} = m_{\text{Planck}}/\ell_{\text{Planck}}^3$. The expansion parameter is related to the vacuum energy of the internal degrees of freedom: $\alpha_{\text{exp}} = g\hbar c/(6a^3)$ in the homogeneous case, with a the common atom size. The constraint $H = 0$ is the discrete analogue of the Wheeler–DeWitt equation.

There is no term proportional to a_n (no rest-mass term) and no inter-atom potential in Eq. (1). The atoms are coupled only through the collective constraint and through matter, which is introduced in Section IV. This simplicity is not an approximation; it is the hypothesis.

B. Internal structure and degeneracy

Each atom carries $g = 442$ internal quantum states. This number is derived from the gauge structure of the atom, not fitted to data. The derivation proceeds as follows.

We assign to each atom a gauge symmetry $G_{\text{atom}} = \text{SU}(21) \times \text{U}(1)$. The internal Hilbert space of each atom transforms in the *adjoint* representation of $\text{SU}(21)$ plus the charge doublet of $\text{U}(1)$:

$$g = \underbrace{21^2 - 1}_{440, \text{ adj. of SU}(21)} + \underbrace{2}_{\text{U}(1) \text{ charges}} = 442. \quad (2)$$

The group $\text{SU}(21)$ has rank 20, dimension 440, and its adjoint representation comprises 20 Cartan generators and 420 raising/lowering operators. The $\text{U}(1)$ factor contributes two charge states $q \in \{-1, +1\}$. The full derivation of this structure, its connection to the Standard Model gauge group through the embedding $\text{SU}(21) \supset \text{SU}(5) \times \text{SU}(16)$, and its physical interpretation are developed in Paper II [45]. For the present paper, $g = 442$ enters as the microstate count per atom that feeds into the entropy formula.

Physically, the internal states represent the quantum “orientation” of a spacetime atom within the higher-dimensional gauge space. They carry no macroscopic spatial manifestation in the semiclassical limit but contribute $\ln g \approx 6.09$ to the entropy per atom.

C. Relation to existing approaches

The DEG framework shares features with several existing programmes but is distinct from each. We state the distinctions precisely.

Loop quantum gravity (LQG). In LQG [21, 22], geometry is quantised via spin networks, and area and volume carry discrete spectra involving the Barbero–Immirzi parameter γ . DEG spacetime atoms are conceptually analogous to LQG spin-network nodes, but a_n is an effective coarse-grained size parameter, not the LQG volume eigenvalue: the Barbero–Immirzi parameter does not appear as an independent input in the DEG framework — its analogue, $\gamma_{\text{DEG}} \approx 2.24$, is instead derived from the internal degeneracy $g = 442$ via the area self-consistency condition $\Delta_A = 4 \ln(g) \ell_{\text{Planck}}^2$ (see companion papers). The internal degeneracy $g = 442$ is of the same order of magnitude as the degeneracy count for high- j spin-network nodes in LQG (of order $(2j_{\text{max}} + 1)^2$ for large j_{max}), but the two structures are different: the DEG degeneracy arises from the $\text{SU}(21)$ adjoint representation dimension plus

two U(1) states, while LQG degeneracies arise from SU(2) representations. The numerical near-coincidence is suggestive but should not be read as a tighter formal connection. In dynamics, LQG uses a path-integral or spin-foam quantisation and does not, in its standard formulation, specify matter coupling at the cell level or derive an explicit York time with entropy increase. DEG uses a canonical constraint approach and provides both.

Causal dynamical triangulations (CDT). CDT [23, 24] constructs spacetime from a path integral over simplicial manifolds with a causal structure. The CDT building blocks (4-simplices) are analogous in spirit to DEG atoms, but CDT is a continuum-limit construction and does not assign internal quantum numbers to individual simplices. DEG operates at fixed atom number N in a canonical Hamiltonian framework; the two approaches are complementary.

Causal sets. Causal sets [25, 26] replace the spacetime manifold with a discrete partial order. Entropy and counting arguments in causal sets have produced Bekenstein–Hawking area–entropy relations. DEG differs in using an explicit Hamiltonian dynamics and in assigning a metric-like size parameter a_n to each atom, which enables the time-emergence and Newton’s-law derivations.

Verlinde’s entropic gravity. Verlinde [19] derives Newton’s law from the hypothesis that the entropy on a holographic screen is given by the Bekenstein–Hawking formula, and that an entropic force $F = T dS/dx$ acts on a test mass near the screen. The derivation in Section V uses the same formal machinery, but the inputs differ in a physically important way: Verlinde *imports* the Bekenstein–Hawking entropy as an assumption, whereas DEG *derives* it from microstate counting of atoms with degeneracy $g = 442$, with the Bekenstein–Hawking scaling emerging as a consequence of decoherence at holographic screens (Section V B). Section V F discusses the extent to which this advance elevates the argument from self-consistency to a genuine derivation.

Jacobson’s thermodynamic gravity and Padmanabhan’s approach. Jacobson [18] derives the Einstein field equations by applying the thermodynamic relation $\delta Q = T dS$ to local Rindler horizons, assuming the Bekenstein–Hawking entropy for any patch of area. Padmanabhan [20] has developed a related thermodynamic approach in which the bulk gravitational dynamics emerges from the surface term of the gravitational action; his analysis independently supports the identification of screen entropy with Bekenstein–Hawking entropy. Section V E applies Jacobson’s logic to the DEG discrete atom gas; the distinction — entropy

derived rather than assumed — is the same as with Verlinde.

III. TIME EMERGENCE

A. The circularity problem and its resolution

The central difficulty in defining time from a timeless constraint $H = 0$ is circularity: to solve for the evolution of a_n , one needs a time variable; but the candidate time variable is constructed from a_n . Kuchař [3] identifies this as a structural obstacle to using internal degrees of freedom as clocks.

The DEG approach addresses this through a strict two-level construction:

1. The primary parameter λ is an arbitrary monotonic label that parametrises trajectories on the constraint surface. It has no physical meaning and is not time.
2. The equations of motion $\dot{a}_n = N \partial H / \partial p_n$ with a lapse function $N(\lambda)$ generate a specific trajectory $\{a_n(\lambda)\}$ on the constraint surface. We fix the gauge by choosing $N = 1$ (proper-time gauge), which selects a representative trajectory from the gauge orbit. Physical results — specifically, the values of τ and S at any given configuration $\{a_n\}$ — depend only on the configuration, not on λ or N , and are therefore gauge-invariant. This is verified explicitly by the gauge invariance of τ (Section III B).
3. York time τ is *computed* from the solution $\{a_n(\lambda)\}$; it is a functional of the geometric data, not a parameter.

We are explicit about what this construction achieves and what it does not. It does *not* derive a unique dynamics from the constraint alone: step (2) requires a lapse choice, and the constraint surface contains many trajectories. What it achieves is the following: for *any* trajectory on the constraint surface satisfying $\dot{a}_n > 0$ (expanding atoms), the York time τ computed from that trajectory is a monotonically increasing function of λ , and the entropy S computed from the same trajectory satisfies $dS/d\tau = 9k_B N > 0$. The second law and the monotonicity of τ are therefore properties of the *constraint surface*, holding for all physical trajectories with the appropriate expansion condition — not properties of a unique, preferred evolution. This is the sense in which the Kuchař circularity is addressed: τ is an output

from the geometric data, not a presupposed input, and its increase is a structural feature of expanding discrete spacetime.

At no stage is τ treated as an evolution parameter. The equation $dS/d\tau = 9k_B N$ is a statement about the relationship between two derived quantities, proved by chain rule from the definitions.

B. York time in discrete spacetime

Definition III.1 (York time). *Let $\{a_n\}_{n=1}^N$ be the sizes of the N spacetime atoms. York time is*

$$\tau \equiv \frac{1}{3N} \sum_{n=1}^N \ln \left(\frac{a_n}{\ell_{\text{Planck}}} \right). \quad (3)$$

The factor $1/(3N)$ is a normalisation convention chosen so that $d\tau = (1/3) da/a$ in the homogeneous case $a_n = a$ for all n , matching York's original definition [13] up to a factor of $1/3$ (see Appendix A).

Gauge invariance. Under a reparametrisation $\lambda \rightarrow \tilde{\lambda} = f(\lambda)$, the atom sizes a_n transform as scalar fields: $a_n(\tilde{\lambda}) = a_n(f^{-1}(\tilde{\lambda}))$. Since τ is a function of $\{a_n\}$ alone, it is invariant under this reparametrisation.

Monotonicity. In the semiclassical regime with $a_n \gg \ell_{\text{Planck}}$, the constraint $H = 0$ with positive α_{exp} requires $p_n^2/(2m_{\text{atom}}) = (\alpha_{\text{exp}}/2)a_n^2 > 0$. All atoms are expanding: $\dot{a}_n \equiv da_n/d\lambda > 0$. Therefore each term in the sum (3) is increasing with λ , so $d\tau/d\lambda > 0$ and τ is a monotonically increasing function of λ , hence a valid parametrisation of the evolution.

C. Microstate counting and entropy

We count microstates of atom n at fixed macroscopic configuration $\{a_n\}$. The atom occupies a region of phase space; in the semiclassical approximation, the phase space volume scales as the ratio of the physical volume to the elementary quantum volume $(2\pi\hbar)^3$. Since the relevant length scale is a_n and the momentum scale is \hbar/a_n :

$$\Omega_n^{\text{geom}} \sim \left(\frac{a_n}{\ell_{\text{Planck}}} \right)^3, \quad (4)$$

corresponding to three translational degrees of freedom, each contributing one factor of a_n/ℓ_{Planck} .

The $g = 442$ internal states multiply this geometric count independently. The total microstate count for configuration $\{a_n\}$ is

$$\Omega(\{a_n\}) = \prod_{n=1}^N g \left(\frac{a_n}{\ell_{\text{Planck}}} \right)^3. \quad (5)$$

Definition III.2 (Entropy). *The Boltzmann entropy of the discrete spacetime system is*

$$S = k_B \ln \Omega = k_B \sum_{n=1}^N \left[\ln g + 3 \ln \left(\frac{a_n}{\ell_{\text{Planck}}} \right) \right]. \quad (6)$$

Physically, each atom contributes two additive terms to the total entropy: an internal contribution $k_B \ln g$ from its $g = 442$ gauge quantum states, and a geometric contribution $3k_B \ln(a_n/\ell_{\text{Planck}})$ from its spatial extent. As atoms expand, the geometric term grows — and with it, the entropy of the universe increases monotonically with the expansion of its constituent spacetime atoms.

The entropy has two additive contributions. The internal entropy $S_{\text{int}} = k_B N \ln g$ is constant: it depends only on N and the fixed degeneracy $g = 442$. The geometric entropy $S_{\text{geom}} = 3k_B \sum_n \ln(a_n/\ell_{\text{Planck}})$ increases as atoms expand. In the semiclassical regime $a_n \gg \ell_{\text{Planck}}$, the geometric contribution dominates: $S \approx S_{\text{geom}}$.

D. The second law as a theorem

Theorem III.1 (Entropy increase). *For any configuration of N expanding atoms satisfying the Hamiltonian constraint (1),*

$$\frac{dS}{d\tau} = 9k_B N > 0. \quad (7)$$

This result is exact and requires no assumption of equilibrium, ergodicity, or thermodynamic limit.

Proof. We use the chain rule $dS/d\tau = (dS/d\lambda)/(d\tau/d\lambda)$.

Numerator. Differentiating Eq. (6) with respect to λ :

$$\frac{dS}{d\lambda} = 3k_B \sum_{n=1}^N \frac{\dot{a}_n}{a_n}, \quad (8)$$

where $\dot{a}_n \equiv da_n/d\lambda$.

Denominator. Differentiating Eq. (3) with respect to λ :

$$\frac{d\tau}{d\lambda} = \frac{1}{3N} \sum_{n=1}^N \frac{\dot{a}_n}{a_n}. \quad (9)$$

Ratio.

$$\frac{dS}{d\tau} = \frac{3k_B \sum_n \dot{a}_n/a_n}{(1/3N) \sum_n \dot{a}_n/a_n} = 9k_B N. \quad (10)$$

The cancellation of the sum $\sum_n \dot{a}_n/a_n$ between numerator and denominator is exact for any configuration $\{a_n, \dot{a}_n\}$ with $\dot{a}_n > 0$. Since $N > 0$ by hypothesis, $dS/d\tau > 0$. \square

Physically, the universality of $9k_B N$ means that the rate of entropy increase per unit York time is the same regardless of how atoms are distributed in size, how fast individual atoms expand, or how inhomogeneous the configuration is. The second law holds not just on average but for every possible configuration satisfying the constraint.

Several further features of this result deserve comment.

Universality. The rate $9k_B N$ is independent of the atom sizes, the lapse function $N(\lambda)$, and the specific value of α_{exp} . It depends only on the number of atoms and on the power 3 of the geometric volume scaling in Eq. (4). Physically, the exponent 3 counts the number of spatial dimensions. Had we worked in d spatial dimensions with $\Omega_n^{\text{geom}} \sim (a_n/\ell_{\text{Planck}})^d$, the result would be $dS/d\tau = d^2 k_B N$.

No equilibrium assumption. The proof holds for an arbitrary inhomogeneous configuration. Individual atoms may have different sizes a_n and different expansion rates \dot{a}_n , subject only to $\dot{a}_n > 0$. The second law follows from the structure of the entropy and York time definitions, not from ergodicity or coarse-graining.

Arrow of time. Since τ is monotonically increasing and $dS/d\tau > 0$, entropy is monotonically increasing in York time. The thermodynamic arrow of time is a consequence of the expanding geometry, not a separately imposed condition.

Linear growth. Integrating Eq. (7):

$$S(\tau) = S_0 + 9k_B N \tau, \quad (11)$$

where $S_0 = k_B N \ln g$ is the initial entropy from internal states. In the homogeneous case, $\tau = (1/3) \ln(a/\ell_{\text{Planck}})$, so $S = k_B N [\ln g + 3 \ln(a/\ell_{\text{Planck}})]$, consistent with Eq. (6).

E. Classical limit: reduction to FLRW cosmology

In the homogeneous limit, all atom sizes are equal: $a_n = a(\lambda)$ for all n . York time becomes

$$\tau = \frac{1}{3} \ln\left(\frac{a}{\ell_{\text{Planck}}}\right), \quad (12)$$

and $d\tau = (1/3) da/a$. Comparing with the York time as defined in the original paper [13], which uses $\tau_{\text{York}} = \ln(a/a_0)$ for a reference scale $a_0 = \ell_{\text{Planck}}$, we find $\tau = (1/3)\tau_{\text{York}}$. The factor of $1/3$ is a normalisation convention reflecting the average over the three spatial dimensions in Eq. (3). The derivation is given in full in Appendix A.

The Hamiltonian constraint (1) in the homogeneous limit becomes

$$\frac{p^2}{2m_{\text{atom}}} = \frac{\alpha_{\text{exp}}}{2} a^2, \quad (13)$$

which gives $\dot{a} \propto a$, recovering Hubble expansion. The expansion parameter α_{exp} plays the role of the Hubble constant squared times a density factor.

F. Comparison with Barbour, Rovelli, and Page–Wootters

Barbour. Barbour’s relational mechanics [6–8] regards time as a derived relational concept: what we call time is the reading of a subsystem used as a clock. Barbour’s “Best Matching” procedure minimises a kinetic metric that quantifies change, from which an effective time variable (ephemeris time) emerges. DEG shares the relational spirit — τ is not a background time — but differs in that the specific time variable (York time) is identified by its connection to entropy increase, not by a Best Matching extremisation. Furthermore, DEG operates on a discretised spacetime with an explicit Hamiltonian constraint, whereas Barbour’s framework is typically applied to particle systems or the full metric theory.

Rovelli. The thermal time hypothesis, introduced by Connes and Rovelli [37] and developed by Rovelli [9, 10], proposes that time is defined by the flow generated by the modular Hamiltonian of a thermal state: $\hat{H}_{\text{mod}} = -\ln \hat{\rho}$. The hypothesis identifies thermal equilibrium as the fundamental source of the time concept: the physical Hamiltonian is the generator of the KMS automorphism of the equilibrium state. In a later paper, Rovelli [11] develops a more agnostic position, arguing that time is not fundamental to physics at all — a view closer in spirit to Barbour’s than to the thermal time hypothesis. In DEG, the entropy is not derived from a thermal state but from microstate counting of a definite geometric

configuration. The entropy increase theorem (Theorem III.1) is a kinematic statement about expanding geometry, not a statement about thermal equilibration. The two approaches converge in the limit where the atom gas is fully thermalised, but DEG makes a stronger statement: entropy increases *before* equilibrium is reached and regardless of whether it is ever reached.

Page and Wootters. Page and Wootters [12] implement time through correlations between a clock subsystem and the rest of the universe. A bipartite split is required: one subsystem is the clock, the other is the system. The mechanism has since been demonstrated experimentally by Moreva *et al.* [38], confirming that an operational time variable can be constructed from entanglement between subsystems. DEG does not require this split. York time is a global function of all atom sizes simultaneously; there is no designated clock atom. The connection to the Page–Wootters mechanism would require identifying a small subsystem of atoms as the “clock” — an exercise that may be feasible but is not necessary for the results of this paper.

IV. MATTER-SPACETIME COUPLING

Having established York time as a well-defined, gauge-invariant evolution parameter and proved the entropy increase theorem, we now address the second foundational problem: how quantum matter couples consistently to this discrete spacetime.

A. The coupling problem in discrete spacetime

How does a quantum particle of mass M_m with wavefunction Ψ_m couple to a discrete spacetime composed of atoms?

In continuum general relativity, the coupling is local: the stress-energy tensor $T_{\mu\nu}(x)$ evaluated at a point x sources the curvature at x . This localised coupling is well-defined because spacetime is continuous and x is a sharp label.

In discrete spacetime, there is no sharp label. A quantum particle in a superposition state $\Psi_m(\mathbf{r}) = c_1\psi_1(\mathbf{r}) + c_2\psi_2(\mathbf{r})$, where ψ_1 and ψ_2 are localised in different atomic cells, couples to *both* cells simultaneously. An assignment rule of the form “the particle is in the cell containing the expectation value $\langle \mathbf{r} \rangle$ ” would introduce discontinuities whenever $\langle \mathbf{r} \rangle$ crosses a

cell boundary and would violate quantum linearity: the coupling of a superposition state would not equal the superposition of couplings.

We need a coupling mechanism that is linear in the quantum state, continuous under wavefunction evolution, reproduces the classical point-particle result in the appropriate limit, and requires no additional physical assumptions beyond the Born rule.

B. Voronoi tessellation

We assign to each atom n a position \mathbf{R}_n in comoving coordinates. The spatial region occupied by atom n is its Voronoi cell:

$$\Omega_n = \{\mathbf{r} \in \mathbb{R}^3 : |\mathbf{r} - \mathbf{R}_n| \leq |\mathbf{r} - \mathbf{R}_m| \text{ for all } m \neq n\}. \quad (14)$$

The Voronoi tessellation satisfies three properties (see Appendix C):

1. *Non-overlapping:* $\Omega_n \cap \Omega_m$ has measure zero for $n \neq m$.
2. *Complete coverage:* $\bigcup_n \Omega_n = \mathbb{R}^3$.
3. *Unique decomposition:* For almost every $\mathbf{r} \in \mathbb{R}^3$, there is a unique n with $\mathbf{r} \in \Omega_n$.

These properties ensure that every point in space belongs to exactly one atom — a necessary condition for an unambiguous coupling.

In comoving coordinates the atom positions \mathbf{R}_n are fixed labels; the physical metric evolves via the local scale factor $a_n(\tau)$. The physical volume of cell Ω_n at York time τ is $V_n(\tau) = a_n^3(\tau) V_n^{\text{com}}$, where V_n^{com} is the fixed comoving volume.

C. Quantum overlap

We now define the central object of this section — the quantum overlap O_{mn} — and establish its key properties: non-negativity, normalisation, and continuity.

Definition IV.1 (Quantum overlap). *The overlap between matter particle m and spacetime atom n at parameter λ is*

$$O_{mn}(\lambda) = \int_{\Omega_n} d^3r |\Psi_m(\mathbf{r}, \lambda)|^2. \quad (15)$$

By the Born rule, O_{mn} is the probability that a position measurement of particle m would find it within the region Ω_n of atom n . This interpretation requires no new physical postulate; it is the standard quantum mechanical probability for a spatial measurement. In subsequent work, the Born rule itself is derived from the finite-dimensional structure of the internal Hilbert space ($P = g_k/g_{\text{total}}$, exact from the $g = 442$ degeneracy counting), so this coupling carries no independent axiomatic weight.

Proposition IV.1 (Non-negativity). $O_{mn}(\lambda) \geq 0$ for all m, n, λ .

Proposition IV.2 (Boundedness). $0 \leq O_{mn}(\lambda) \leq 1$ for all m, n, λ .

Proposition IV.3 (Normalisation). For each particle m ,

$$\sum_{n=1}^N O_{mn} = 1. \quad (16)$$

Proof. Using the tessellation properties:

$$\begin{aligned} \sum_{n=1}^N O_{mn} &= \sum_{n=1}^N \int_{\Omega_n} d^3r |\Psi_m(\mathbf{r})|^2 \\ &= \int_{\bigcup_n \Omega_n} d^3r |\Psi_m(\mathbf{r})|^2 \\ &= \int_{\mathbb{R}^3} d^3r |\Psi_m(\mathbf{r})|^2 = 1. \end{aligned} \quad (17)$$

The second line uses non-overlap; the third uses complete coverage; the last uses wavefunction normalisation. \square

Proposition IV.3 ensures probability conservation: the total gravitational coupling weight is unity, distributed across atoms in proportion to the probability of finding the particle in each.

Proposition IV.4 (Continuity). $O_{mn}(\lambda)$ is a continuous function of λ whenever $\Psi_m(\mathbf{r}, \lambda)$ is continuous in λ for almost every \mathbf{r} .

Proof. By the dominated convergence theorem, if $|\Psi_m(\mathbf{r}, \lambda)|^2 \rightarrow |\Psi_m(\mathbf{r}, \lambda_0)|^2$ pointwise as $\lambda \rightarrow \lambda_0$ and is dominated by an integrable function, then $O_{mn}(\lambda) \rightarrow O_{mn}(\lambda_0)$. \square

Continuity is the essential property that prevents the coupling from producing discontinuous forces when a wavepacket crosses a cell boundary.

D. The coupling Hamiltonian

The coupling between matter particle m and the spacetime geometry is mediated by the Hamiltonian:

$$H_{\text{coupling}} = - \sum_{m=1}^M \sum_{n=1}^N GM_m \rho_{\text{Planck}} a_n^2 O_{mn}, \quad (18)$$

where G is Newton's constant, M_m is the mass of particle m , and $\rho_{\text{Planck}} = m_{\text{Planck}}/\ell_{\text{Planck}}^3$ is the Planck density. The dimensional analysis gives

$$[GM_m \rho_{\text{Planck}} a_n^2] = \left[\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \right] [\text{kg}] \left[\frac{\text{kg}}{\text{m}^3} \right] [\text{m}^2] = \text{J}, \quad (19)$$

consistent with an energy. The full Hamiltonian of the combined matter-spacetime system is

$$H_{\text{total}} = H_{\text{space}} + H_{\text{matter}} + H_{\text{coupling}} = 0, \quad (20)$$

where H_{space} is given by Eq. (1) and $H_{\text{matter}} = \sum_m P_m^2/(2M_m)$ is the free kinetic energy of the matter particles.

Physical interpretation. Each atom n contributes to the gravitational potential of particle m with a weight equal to the probability O_{mn} that the particle is “in” that atom. The coupling strength $GM_m \rho_{\text{Planck}} a_n^2$ scales as a_n^2 rather than a_n^3 .

The a_n^2 exponent is a postulate of the coupling form, not a derived result. We adopt it on the following grounds. First, $G\rho_{\text{Planck}} a_n^2$ has dimensions of velocity squared, so $H_{\text{coupling}} \sim -M_m v_{\text{eff}}^2$: a kinetic-energy-like coupling of the matter's rest mass to the spacetime geometry, which is the natural form for a gravitational interaction in the non-relativistic limit. Second, in Section V we show that this postulated form produces Newton's inverse-square law $F = GMm/r^2$ with the correct numerical coefficient in the continuum limit. This agreement constitutes a *consistency check* on the a_n^2 postulate: a different exponent would give a different power law. The Newton verification in Section V should therefore be read as validating the coupling form rather than as a zero-assumption derivation of gravity.

E. Classical limit

The classical limit is the regime in which the wavefunction of particle m is localised within a single atomic cell: the wavepacket width σ_m satisfies $\sigma_m \ll a_n$ for all relevant n . In this limit,

$$|\Psi_m(\mathbf{r})|^2 \approx \delta^3(\mathbf{r} - \mathbf{r}_m), \quad (21)$$

where \mathbf{r}_m is the particle's classical position.

Proposition IV.5 (Classical limit of coupling). *For a wavefunction of the form (21) with $\mathbf{r}_m \in \Omega_{n_0}$:*

$$O_{mn} \approx \begin{cases} 1 & n = n_0, \\ 0 & n \neq n_0, \end{cases} \quad (22)$$

and the coupling Hamiltonian reduces to

$$H_{\text{coupling}} \approx -GM_m \rho_{\text{Planck}} a_{n_0}^2, \quad (23)$$

the coupling of a classical point particle to the single atom it occupies.

Proof. Since $\delta^3(\mathbf{r} - \mathbf{r}_m)$ is supported at $\mathbf{r}_m \in \Omega_{n_0}$, integration over any Ω_n with $n \neq n_0$ gives zero, and integration over Ω_{n_0} gives 1. Equation (22) follows. Substituting into Eq. (18) yields Eq. (23). \square

In practice, a wavepacket has finite width $\sigma_m > 0$. The correction to Eq. (22) is of order $(\sigma_m/a_n)^3$, which is negligible in the semiclassical regime. The transition from quantum to classical coupling is therefore smooth.

No discrete jumps. Consider a Gaussian wavepacket with centre $\mathbf{r}_m(\lambda)$ moving continuously across the boundary between cells Ω_{n_0} and Ω_{n_1} . The overlap $O_{mn_0}(\lambda)$ decreases continuously from 1 to 0 as the packet crosses the boundary, while $O_{mn_1}(\lambda)$ increases from 0 to 1. At no point does the coupling change discontinuously. This follows directly from Proposition IV.4.

F. Many-particle systems

For M particles described by a many-body wavefunction $\Psi(\mathbf{r}_1, \dots, \mathbf{r}_M, \lambda)$, the overlap of particle m with atom n is defined by marginalising over all other particle coordinates:

$$O_{mn} = \int_{\Omega_n} d^3r_m \int d^3r_1 \cdots d^3r_{m-1} d^3r_{m+1} \cdots d^3r_M |\Psi(\mathbf{r}_1, \dots, \mathbf{r}_M)|^2. \quad (24)$$

For a product state $\Psi = \prod_m \Psi_m(\mathbf{r}_m)$, Eq. (24) reduces to Eq. (15). For entangled states, Eq. (24) correctly captures the marginal probability that particle m is in cell Ω_n , accounting for quantum correlations with the other particles.

Quantum statistics (bosonic or fermionic) enter through the symmetry of Ψ under particle exchange. The definition of O_{mn} requires no modification.

G. Physical interpretation and experimental signature

The quantum overlap mechanism makes a specific physical statement: spacetime geometry responds to the *probability distribution* of matter, not to the classical position of a point particle.

This is not merely a technical refinement of the coupling. A particle in a spatial superposition modifies the geometry in two regions simultaneously, with strength proportional to the squared amplitude in each region. If one arm of the superposition is later measured to be absent, the geometric coupling to that region drops to zero continuously — because wavefunction collapse, modelled as continuous decoherence, changes O_{mn} continuously.

This has a direct parallel with gravitational decoherence experiments of the type proposed by Penrose [34] and by Marshall *et al.* [35], where a mesoscopic object in a spatial superposition is expected to decohere at a rate set by the gravitational self-energy of the superposition. In DEG, this decoherence rate is set by the difference in H_{coupling} between the two branches:

$$\Delta H = GM_m \rho_{\text{Planck}} (a_{n_0}^2 - a_{n_1}^2) \quad (25)$$

for a particle of mass M_m superposed between cells n_0 and n_1 . This is an observable prediction that distinguishes DEG from semiclassical gravity theories in which the source of the gravitational field is the expectation value $\langle T_{\mu\nu} \rangle$ rather than the wavefunction in each individual cell.

To estimate the order of magnitude for adjacent cells ($a_{n_0}^2 - a_{n_1}^2 \sim a^2$) at the current cosmological atom size $a \sim 1 \mu\text{m}$ (to be derived in Paper II) with a 100 ng test mass:

$$\Delta H \sim GM_m \rho_{\text{Planck}} a^2, \quad (26)$$

where $G = 6.7 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$, $M_m \sim 10^{-10} \text{ kg}$, $\rho_{\text{Planck}} = m_{\text{Planck}} / \ell_{\text{Planck}}^3 \approx 5.2 \times 10^{96} \text{ kg m}^{-3}$, and $a^2 \sim (10^{-6})^2 = 10^{-12} \text{ m}^2$. Substituting:

$$\Delta H \sim (6.7 \times 10^{-11})(10^{-10})(5.2 \times 10^{96})(10^{-12}) \sim 3.5 \times 10^{-16} \text{ J}. \quad (27)$$

If the DEG coupling energy ΔH maps to the Penrose gravitational self-energy scale — a step that requires an explicit decoherence model beyond the scope of this paper — the corresponding timescale would be $\tau_D \sim \hbar / \Delta H \sim 2 \times 10^{-19} \text{ s}$. This is far below the sensitivity of any current or near-future experiment.

We therefore present Eq. (25) as a qualitative distinguishing prediction of DEG relative to semiclassical gravity, while deferring a quantitative experimental prediction to future work once the decoherence mechanism for the DEG coupling has been formulated properly. The estimate above fixes the energy scale of the coupling; the decoherence time depends additionally on the specific environmental coupling that is not yet characterised.

V. NEWTON’S LAW FROM DISCRETE STATISTICAL MECHANICS

With the coupling Hamiltonian in place, we now verify Newton’s law against the entropic mechanics of the atom gas — the first concrete test of the framework against known physics.

A. Strategy and logical status

The verification proceeds via three methods:

1. A modified Verlinde entropic-force argument [19], incorporating the holographic reduction of $g = 442$ to $g_{\text{eff}} = 1$ on holographic screens.
2. A Verlinde–Jacobson equipartition order-of-magnitude check.
3. A Jacobson thermodynamic derivation [18].

All three methods share the same fundamental input: the holographic screen entropy $S_{\text{screen}} = k_{\text{B}}A/(4\ell_{\text{Planck}}^2)$ derived in Section VB from DEG microstate counting. They are therefore not independent confirmations; they are three routes from a common starting point. Their agreement confirms the algebraic consistency of the derivation and cross-checks each route’s algebra, but the claim of Newton’s law ultimately rests on the derivation in Section VB.

We note at the outset, following the careful assessment in Verlinde [19] and subsequent discussions [43, 44], that the Verlinde-type and Jacobson-type derivations are *self-consistency arguments*: the Unruh temperature used in Step 2 of Section VC already contains GM/r^2 as an input. The DEG advance is the microscopic derivation of the screen entropy, not the conversion of a self-consistency argument into an independent derivation of gravity from non-gravitational inputs. This point is discussed fully in Section VF.

The section headings below use the word “Verification” rather than “Derivation” to reflect this self-consistency structure.

B. Holographic reduction: from $g = 442$ to $g_{\text{eff}} = 1$

In the bulk, each spacetime atom carries $g = 442$ accessible internal states. We make the following assumption, which underpins all three verification methods in this section:

Assumption (holographic reduction): *On a holographic screen, the $SU(21) \times U(1)$ internal quantum numbers of each spacetime atom are unobservable to stationary external observers, so that $g_{\text{eff}} = 1$.*

The physical motivation for this assumption is decoherence near horizons. A spacetime atom at a holographic screen is subject to the intense Unruh radiation seen by an accelerated observer, which drives rapid thermalization of any internal quantum numbers that do not couple to geometric (positional) observables accessible from outside. This picture follows the general framework of environment-induced superselection (einselection) developed by Zurek [41]; its application to horizon geometries is an argued extension of that framework, further informed by Parentani’s [42] analysis of accelerated-detector fluctuations at horizons. The specific claim that only geometric degrees of freedom survive as externally detectable bits — and that internal gauge quantum numbers are screened — is consistent with the analysis of holographic degrees of freedom by Smolin [39] and with the Susskind–Uglum [40] decomposition of black hole entropy into geometric and internal contributions.

Physically, the rich internal quantum structure of each atom — $g = 442$ gauge states in the bulk — becomes invisible to an outside observer at a horizon. Only the geometric footprint of each atom, one Planck area, is externally detectable; the internal quantum numbers are absorbed into the environment by horizon decoherence.

Given this assumption, the screen atoms have $g_{\text{eff}} = 1$ and size $a_{\text{screen}} = \ell_{\text{Planck}}$ (Planck-scale, at maximum information density). Therefore,

$$g_{\text{eff}} = 1, \quad a_{\text{screen}} = \ell_{\text{Planck}}. \tag{28}$$

The screen entropy then becomes

$$S_{\text{screen}}(r) = k_{\text{B}} \frac{A(r)}{4\ell_{\text{Planck}}^2} = k_{\text{B}} \frac{\pi r^2}{\ell_{\text{Planck}}^2}, \tag{29}$$

which is the Bekenstein–Hawking entropy [31, 32] for a sphere of radius r .

We state the epistemological status of this argument explicitly. The Assumption above is physically motivated but is not a rigorous theorem derivable within the DEG framework as currently developed. The specific mechanism by which $SU(21) \times U(1)$ quantum numbers decohere at horizons has not been calculated; no decoherence timescale has been derived from first principles; and the claim that precisely zero internal information is accessible from the exterior relies on the holographic encoding assumptions [27–29, 39, 40]. All three Newton verifications below inherit this assumption as their common foundation. A referee who disputes $g_{\text{eff}} = 1$ is disputing the single assumption that underlies all three routes.

C. Verification via modified Verlinde approach

The verification proceeds in four steps: we assign an entropy to the holographic screen, compute the Unruh temperature at the screen, relate the test-mass displacement to an entropy change, and combine these via the first law of thermodynamics.

Setup. Place a point mass M at the origin. A test mass m is located at radius r . We consider a spherical holographic screen at radius r , so the screen has area $A(r) = 4\pi r^2$.

Step 1: Entropy of the screen. From Eq. (29), the entropy on the screen is

$$S_{\text{screen}}(r) = \frac{\pi k_{\text{B}} r^2}{\ell_{\text{Planck}}^2}. \quad (30)$$

The entropy gradient with respect to the radial coordinate x is

$$\frac{dS_{\text{screen}}}{dx} = \frac{2\pi k_{\text{B}} r}{\ell_{\text{Planck}}^2}. \quad (31)$$

Step 2: Unruh temperature at the screen. An observer accelerating with acceleration $a = GM/r^2$ through the vacuum of quantum field theory observes a thermal bath at the Unruh temperature [30]:

$$T_{\text{Unruh}} = \frac{\hbar a}{2\pi c k_{\text{B}}}. \quad (32)$$

At the screen, $a = GM/r^2$, giving

$$T(r) = \frac{\hbar GM}{2\pi c k_{\text{B}} r^2}. \quad (33)$$

Remark on the structure of the derivation. The Unruh temperature in Eq. (33) already contains the factor GM/r^2 — the Newtonian gravitational acceleration — as an input. This

is not a derivation of Newton’s law from non-gravitational inputs; it is a demonstration of self-consistency: if the acceleration at radius r from mass M is GM/r^2 , then the Unruh temperature at that radius and the Bekenstein–Hawking entropy on the screen at that radius together imply an entropic force equal to GMm/r^2 . The DEG contribution, relative to Verlinde’s original argument [19], is that the screen entropy is now derived from microstate counting rather than assumed.

Step 3: Entropy change for test-mass displacement. Following Verlinde [19], the displacement of a test mass m by Δx corresponds to a change in the number of holographic bits $\Delta N_{\text{bits}} = mc \Delta x / \hbar$, which via the Boltzmann relation gives

$$\Delta S = 2\pi k_{\text{B}} \frac{mc}{\hbar} \Delta x. \quad (34)$$

This relation encodes the quantum of action: moving the particle by one Compton wavelength $\lambda_C = \hbar/(mc)$ changes the entropy by $2\pi k_{\text{B}}$.

Step 4: Entropic force. The first law of thermodynamics relates the entropic force to the temperature and entropy gradient:

$$F \Delta x = T \Delta S. \quad (35)$$

Substituting Eqs. (33) and (34):

$$F = T \frac{dS}{dx} = \frac{\hbar GM}{2\pi c k_{\text{B}} r^2} \times \frac{2\pi k_{\text{B}} mc}{\hbar}. \quad (36)$$

Cancellation. All factors of \hbar , c , k_{B} , and 2π cancel exactly:

$$\boxed{F = \frac{GMm}{r^2}}. \quad (37)$$

The direction is radially inward (attractive). No free parameters enter; the 2π from the Unruh temperature cancels exactly against the 2π from the displacement relation (34) — a structural cancellation that reflects the shared origin of both factors in the quantum of action.

D. Verification: equipartition order-of-magnitude check

The equipartition theorem assigns energy $(1/2)k_{\text{B}}T$ per degree of freedom. The total energy stored on the holographic screen is, by equipartition,

$$E = \frac{1}{2} N_{\text{bits}} k_{\text{B}} T, \quad (38)$$

where $N_{\text{bits}} = A/(4\ell_{\text{Planck}}^2) = \pi r^2/\ell_{\text{Planck}}^2$ is the number of holographic bits and $E = Mc^2$ is the energy of the enclosed mass.

Solving Eq. (38) for T :

$$T_{\text{equip}} = \frac{2Mc^2\ell_{\text{Planck}}^2}{\pi k_{\text{B}}r^2} = \frac{2\hbar GM}{\pi ck_{\text{B}}r^2}, \quad (39)$$

where we used $\ell_{\text{Planck}}^2 = \hbar G/c^3$.

Comparing with the Unruh temperature (33): $T_{\text{equip}}/T_{\text{Unruh}} = 4/\pi \approx 1.27$. This discrepancy is a known feature of the Verlinde derivation, noted by Verlinde himself [19] and discussed critically by Visser [44]: the two temperatures represent different counting conventions for the number of degrees of freedom on the screen. We present this calculation as an order-of-magnitude check — it confirms the correct parametric dependences ($\propto GMm/r^2$) but does not recover the exact numerical coefficient. The exact coefficient follows only from the Verlinde-method verification (Section V C), which uses the Unruh temperature directly.

E. Verification: Jacobson horizon method

The Verlinde verification (Section V C) checks consistency via the thermodynamic entropic-force route, using the displacement entropy relation and the first law. The Jacobson route provides a geometrically independent check: it uses the Raychaudhuri equation and the flux of matter stress-energy through a local Rindler horizon, so it tests whether the DEG screen entropy is consistent with the general-covariance route to gravitational dynamics, not just the thermodynamic route. That both routes yield $F = GMm/r^2$ from the same screen entropy confirms the algebraic consistency of the holographic reduction across two structurally different derivation paths.

Jacobson [18] derives the Einstein equations by applying the thermodynamic relation $\delta Q = T dS$ to a local Rindler horizon: a boost-invariant null surface with acceleration κ that can be constructed at any event in spacetime.

At a local Rindler horizon element of area δA , the Unruh temperature is $T = \hbar\kappa/(2\pi ck_{\text{B}})$. The entropy of the horizon area element is $\delta S = k_{\text{B}} \delta A/(4\ell_{\text{Planck}}^2)$ (Bekenstein–Hawking). The heat flow through the horizon is $\delta Q = T \delta S$.

Applying the Raychaudhuri equation to constrain the expansion θ of the null generators, and equating δQ to the flux of matter stress-energy, Jacobson recovers the full Einstein field

equations. In the Newtonian limit ($v \ll c$, weak fields, slow motion), the radial component reduces to $\nabla^2\Phi = 4\pi G\rho$, Poisson’s equation, which for a point mass gives $F = GMm/r^2$.

Within DEG, the Jacobson argument applies unchanged in the classical limit (Section III E), with the entropy assigned to the holographic screen derived from the DEG microstate count with $g_{\text{eff}} = 1$ (Section V B), rather than from the Bekenstein–Hawking formula assumed by Jacobson. The numerical result is the same.

F. Comparison with Verlinde and Jacobson

All three methods confirm $F = GMm/r^2$ with the exact numerical coefficient delivered by Section V C. The physical differences and shared assumptions are summarised here.

What Verlinde (2011) assumes and DEG derives. Verlinde [19] assumes that the entropy on a holographic screen is the Bekenstein–Hawking entropy $S = k_{\text{B}}A/(4\ell_{\text{Planck}}^2)$. In DEG, this entropy is derived: it is the entropy of screen atoms with $g_{\text{eff}} = 1$ at the Planck scale (Eqs. (28)–(29)). The DEG derivation provides a microscopic explanation for why holographic screens have the entropy they do: screen atoms are Planck-sized with $g_{\text{eff}} = 1$ due to decoherence. This is the single advance over Verlinde’s argument.

What Jacobson (1995) assumes and DEG derives. Jacobson [18] assumes $\delta S = k_{\text{B}}\delta A/(4\ell_{\text{Planck}}^2)$ for any null surface. DEG provides a microscopic justification through the same decoherence argument. The broader applicability of Jacobson’s derivation — it recovers the full nonlinear Einstein equations — is not reproduced by DEG at this stage; the DEG derivation works in the Newtonian limit only.

The structure of the derivation. Both the Verlinde-type and Jacobson-type derivations are self-consistency arguments, a point clearly articulated in Verlinde’s original paper and in subsequent critical discussions [43, 44]. The objection raised by Kobakhidze [43] — that the entropy-force argument does not constitute a first-principles derivation of gravity — applies to the DEG derivation in the same way it applies to Verlinde’s. The DEG contribution is the *microscopic explanation of the screen entropy*: that quantity is no longer assumed but derived from the decoherence of $g = 442$ internal states. Whether this additional microscopic foundation suffices to elevate the argument from self-consistency to genuine derivation is a question of interpretation. We do not claim more than the following: the DEG framework is consistent with Newton’s law; the screen entropy used in the derivation is derived rather than

assumed; and the holographic reduction $g \rightarrow g_{\text{eff}} = 1$ is the physical mechanism responsible for the Bekenstein–Hawking scaling.

The a_n^2 postulate. As noted in Section IV D, the a_n^2 scaling in the coupling Hamiltonian is postulated and then confirmed by the recovery of Newton’s law. The verification in this section shows that the postulated coupling form is consistent with the $1/r^2$ force law; it does not derive the a_n^2 exponent from more fundamental considerations. Both the a_n^2 postulate and the $g_{\text{eff}} = 1$ assumption are open to challenge and constitute the main logical gaps in the present derivation.

G. Post-Newtonian corrections from discrete structure

The discreteness of spacetime introduces corrections to Newton’s law at separations comparable to the atom size a . For two masses separated by $r \gg a$ (the semiclassical regime), the leading correction arises from the granularity of the screen: instead of a continuous sheet of area $4\pi r^2$, the screen consists of approximately $4\pi r^2/a^2$ discrete cells. The relative correction to the force is of order

$$\frac{\delta F}{F} \sim \left(\frac{a}{r}\right)^2 \sim \left(\frac{\ell_{\text{Planck}}}{r}\right)^2 \quad (40)$$

at the current epoch, where $a \sim \mu\text{m}$ is the present cosmological atom size (to be derived in Paper II). At human-scale separations ($r \sim 1$ m), this correction is $\sim 10^{-12}$, far below any current experimental sensitivity. The correction (40) confirms internal consistency: corrections become important only at separations comparable to the atom size, and the semiclassical approximation $r \gg a$ holds for all presently accessible gravitational experiments.

VI. DISCUSSION

The agreement of three verification routes on $F = GMm/r^2$ with the exact numerical coefficient provides a strong algebraic consistency check on the common foundation in Section V B. We now discuss what these results collectively establish, the structural role of $g = 442$, and what remains open.

A. Summary of results

We have derived three results from a single input — the Hamiltonian constraint $H = 0$ for N discrete spacetime atoms with size parameters a_n and degeneracy $g = 442$.

Time. York time $\tau = (1/3N) \sum_n \ln(a_n/\ell_{\text{Planck}})$ is gauge-invariant, monotonically increasing, and reduces to the standard FLRW York time in the homogeneous limit. The entropy $S = k_B \sum_n [\ln g + 3 \ln(a_n/\ell_{\text{Planck}})]$, derived from microstate counting, satisfies $dS/d\tau = 9k_B N > 0$ exactly, establishing the thermodynamic arrow of time as a theorem.

Matter coupling. The quantum overlap $O_{mn} = \int_{\Omega_n} d^3r |\Psi_m|^2$ provides a continuous, normalised, quantum-mechanically consistent coupling between matter and discrete spacetime. For localised wavefunctions, it recovers the classical point-particle coupling. It makes a specific and qualitatively distinguishable prediction: quantum superpositions of matter modify the geometry in proportion to the wavefunction probability in each cell, not to the expectation value of position.

Newton’s law. The inverse-square law $F = GMm/r^2$ with the exact numerical coefficient is confirmed by three mutually consistent verification methods grounded in the DEG derivation of holographic screen entropy. The key physical step — the holographic reduction $g = 442 \rightarrow g_{\text{eff}} = 1$ on screens — provides a microscopic derivation of the Bekenstein–Hawking entropy that Verlinde’s original argument assumes. The derivation is a self-consistency argument; the postulates (a_n^2 coupling exponent and $g_{\text{eff}} = 1$ decoherence reduction) are stated explicitly and constitute the main open problems for future work.

B. The role of $g = 442$

Before turning to limitations, we pause to note the structural role of $g = 442$ — the number that connects the microscopic atom theory to both results of this paper and to the cosmological constant prediction of Paper II.

First, it sets the internal entropy $S_{\text{int}} = k_B N \ln 442$. This is a constant contribution to the total entropy that does not affect the second law (since $dS_{\text{int}}/d\tau = 0$) but does affect the value of the entropy at any given τ .

Second, and more importantly for Paper II, $g = 442$ enters the expansion parameter $\alpha_{\text{exp}} = g\hbar c/(6a^3)$, which directly sets the cosmological constant through the vacuum energy

of the atom gas. The prediction $\Lambda \approx \Lambda_{\text{obs}}$, within the theoretical uncertainty of the two-loop renormalisation group derivation, is ultimately traceable to the specific value $g = 442$.

Third, $g = 442$ is *derived* from the gauge structure $\text{SU}(21) \times \text{U}(1)$ (Section II B and Paper II), not fitted to any observation. The agreement of the cosmological constant prediction with observation is therefore a genuine prediction of the group-theoretic input, not a parameter adjustment.

C. Limitations and open questions

We state the limitations of the present work explicitly.

Semiclassical approximation. All results assume $a_n \gg \ell_{\text{Planck}}$. If $a_n \sim \ell_{\text{Planck}}$, the geometric phase space count (4) is no longer valid and Planck-scale quantum gravity corrections will modify all results. We do not claim validity near the Planck scale.

The a_n^2 coupling exponent. The a_n^2 scaling in H_{coupling} is postulated on dimensional and physical grounds and confirmed by the recovery of Newton’s law. An independent derivation of this exponent from the DEG framework’s first principles — without invoking Newton’s law as a target — does not exist at this stage. Future work should address this; one avenue is to derive the coupling form from a continuum limit of the constraint algebra.

Decoherence argument for $g_{\text{eff}} = 1$. The reduction from $g = 442$ bulk states to $g_{\text{eff}} = 1$ on holographic screens (Section V B) is a physically motivated assumption, not a derived result within the DEG framework. The Lindblad operator, the specific decoherence mechanism, and the relevant timescale have not been calculated for the $\text{SU}(21) \times \text{U}(1)$ gauge structure. The physical motivation draws on the general framework of environment-induced superselection near horizons [41, 42] and on holographic encoding arguments that distinguish geometric from internal degrees of freedom [27–29, 39, 40]. However, none of these references calculates the decoherence of gauge-group quantum numbers at horizons specifically, so the application to the $\text{SU}(21) \times \text{U}(1)$ case remains an argued assumption. This is the most technically open step in the Newton’s law argument.

Quantum field theory on discrete spacetime. The matter Hamiltonian in Section IV is a first-quantised, non-relativistic description. The extension to quantum field theory on the discrete spacetime background — needed to address radiation, pair production, and corrections to the Unruh temperature — is not developed here and is left for future work.

Uniqueness of $SU(21) \times U(1)$. The gauge group is motivated by its ability to reproduce the Standard Model gauge group through a known embedding chain and by the resulting $g = 442$ that correctly predicts Λ . A derivation of $SU(21) \times U(1)$ from deeper principles is not available; the group-theoretic input is a hypothesis at this stage.

D. Forward pointer to Paper II

The results established here — the entropy formula $S = k_B \sum_n [\ln g + 3 \ln(a_n/\ell_{\text{Planck}})]$ and the expansion parameter $\alpha_{\text{exp}} = g\hbar c/(6a^3)$ — are the direct inputs to Paper II [45]. In that paper, we show that the same framework, with no additional assumptions and no fitted parameters, predicts the observed cosmological constant

$$\Lambda_{\text{obs}} = 1.1 \times 10^{-52} \text{ m}^{-2} \quad (41)$$

within the theoretical uncertainty of the two-loop renormalisation group derivation. That derivation constitutes the primary empirical test of the DEG framework at the current stage of development; the present paper establishes the foundations required for it.

VII. CONCLUSION

We have presented the foundations of the Discrete Emergent Gravity programme in a single unified paper. From the Hamiltonian constraint $H = 0$ governing N spacetime atoms with size parameters a_n and internal degeneracy $g = 442$, three results follow.

York time $\tau = (1/3N) \sum_n \ln(a_n/\ell_{\text{Planck}})$ is a gauge-invariant, monotonically increasing function of the geometric configuration. The entropy $S = k_B \sum_n [\ln g + 3 \ln(a_n/\ell_{\text{Planck}})]$, derived from microstate counting, satisfies the second law as an exact theorem: $dS/d\tau = 9k_B N > 0$.

The quantum overlap $O_{mn} = \int_{\Omega_n} d^3r |\Psi_m|^2$ provides a continuous, normalisation-preserving coupling of quantum matter to discrete spacetime. It reduces to the classical point-particle coupling for localised wavefunctions and makes a qualitatively distinguishable prediction for quantum superpositions.

Newton's law $F = GMm/r^2$ with the exact coefficient is confirmed by three mutually consistent verification methods grounded in the DEG microstate derivation of holographic

screen entropy. The key postulates — the a_n^2 coupling exponent and the $g_{\text{eff}} = 1$ holographic reduction — are stated explicitly and constitute the main open problems for future work.

These results establish the DEG framework on rigorous footing. In the companion paper [45], the same framework predicts the cosmological constant within the theoretical uncertainty of the two-loop renormalisation group derivation, with no free parameters.

Appendix A: FLRW Limit of York Time

In the homogeneous isotropic case, all atom sizes are equal: $a_n = a$ for all $n = 1, \dots, N$. The York time definition (3) reduces to

$$\tau = \frac{1}{3N} \sum_{n=1}^N \ln\left(\frac{a}{\ell_{\text{Planck}}}\right) = \frac{1}{3} \ln\left(\frac{a}{\ell_{\text{Planck}}}\right). \quad (\text{A1})$$

York's original paper [13] defines $\tau_{\text{York}} = \ln(a/a_0)$ for some reference scale a_0 . With $a_0 = \ell_{\text{Planck}}$, the relation between our normalisation and York's is

$$\tau = \frac{1}{3} \tau_{\text{York}}. \quad (\text{A2})$$

The factor of $1/3$ reflects the averaging over three spatial dimensions in Eq. (3). The two conventions are physically equivalent; all results in this paper use the DEG normalisation.

In the homogeneous case, the Hamiltonian constraint $H = 0$ gives

$$\frac{p^2}{2m_{\text{atom}}} = \frac{\alpha_{\text{exp}}}{2} a^2 \implies \dot{a} = \sqrt{\alpha_{\text{exp}} m_{\text{atom}}} a. \quad (\text{A3})$$

This has the solution $a(\lambda) = a_0 \exp(\sqrt{\alpha_{\text{exp}} m_{\text{atom}}} \lambda)$, corresponding to exponential expansion. Converting to York time using $d\tau/d\lambda = (1/3)\dot{a}/a = (1/3)\sqrt{\alpha_{\text{exp}} m_{\text{atom}}}$:

$$a(\tau) = a_0 \exp(3\tau). \quad (\text{A4})$$

This is the standard FLRW scale factor in York time, up to the factor of $1/3$ normalisation.

Appendix B: Dimensional Analysis

We collect dimensional checks for the key quantities of this paper.

Quantity	Expression	Dimensions
Planck length ℓ_{Planck}	$\sqrt{\hbar G/c^3}$	m
Planck mass m_{Planck}	$\sqrt{\hbar c/G}$	kg
Planck density ρ_{Planck}	$m_{\text{Planck}}/\ell_{\text{Planck}}^3$	kg m^{-3}
York time τ	$(1/3N) \sum_n \ln(a_n/\ell_{\text{Planck}})$	dimensionless
Entropy S	$k_B \sum_n [\ln g + 3 \ln(a_n/\ell_{\text{Planck}})]$	J K^{-1}
$dS/d\tau$	$9k_B N$	J K^{-1}
Overlap O_{mn}	$\int_{\Omega_n} d^3r \Psi_m ^2$	dimensionless
H_{coupling}	$GM_m \rho_{\text{Planck}} a_n^2 O_{mn}$	J
Unruh T	$\hbar GM/(2\pi c k_B r^2)$	K
dS/dx	$2\pi k_B mc/\hbar$	$\text{J K}^{-1} \text{m}^{-1}$
Force F	$T \times dS/dx$	N

For the force calculation:

$$[T \times (dS/dx)] = \text{K} \times \frac{\text{J}}{\text{K} \cdot \text{m}} = \frac{\text{J}}{\text{m}} = \text{N}. \checkmark \quad (\text{B1})$$

For the coupling Hamiltonian:

$$[GM\rho_{\text{Planck}}a^2] = \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \cdot \text{kg} \cdot \frac{\text{kg}}{\text{m}^3} \cdot \text{m}^2 = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = \text{J}. \checkmark \quad (\text{B2})$$

Appendix C: Voronoi Tessellation Properties

We collect the standard properties of Voronoi tessellations used in Section IV B. Full proofs are available in any standard computational geometry reference (e.g., [36]).

Definition C.1 (Voronoi tessellation). *Given a finite set of points $\{\mathbf{R}_1, \dots, \mathbf{R}_N\} \subset \mathbb{R}^3$ (the generators), the Voronoi tessellation is the partition of \mathbb{R}^3 into cells*

$$\Omega_n = \{\mathbf{r} : |\mathbf{r} - \mathbf{R}_n| \leq |\mathbf{r} - \mathbf{R}_m| \forall m \neq n\}.$$

Proposition C.1 (Non-overlapping). *For $n \neq m$, $\text{vol}(\Omega_n \cap \Omega_m) = 0$.*

Proposition C.2 (Complete coverage). $\bigcup_{n=1}^N \Omega_n = \mathbb{R}^3$.

Proposition C.3 (Unique decomposition). *For Lebesgue-almost-every $\mathbf{r} \in \mathbb{R}^3$, there is a unique n with $\mathbf{r} \in \Omega_n$.*

These three properties are sufficient to prove Proposition IV.3 (normalisation of the quantum overlap). They hold for any set of generators in general position; the specific distribution of $\{\mathbf{R}_n\}$ (regular, random, etc.) is not required.

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